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Economic Growth with Tourism and Environmental Change

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Abstract:

The main purpose of this study is to examine dynamic interactions between economic growth, environmental change, and tourism. Although tourism is playing an increasingly important role in different economies, there are only a few theoretical models to dynamic economic and environmental issues with endogenous tourism. On the basis of the Solow-Uzawa growth model, the neoclassical growth model with environmental change, and ideas from tourism economics, we develop a three-sector growth model. The industrial and service sectors are perfectly competitive. The environment sector is financially supported by the government. We introduce taxes not only on producers, but also on consumers' incomes from wage, land, and interest of wealth, consumption of goods and services, and housing. We simulate the motion of the national economy and examine effects of changes in some parameters. The comparative dynamic analysis with regard to the rate of interest, the price elasticity of tourism, the global economic condition, the total productivity of the service sectors, and the propensity to save provides some important insights into the complexity of open economies with endogenous wealth and environment.

Keywords: *Tourism, Price elasticity of tourism, Wealth accumulation, Environmental change, Elastic labor supply*

JEL Code: O41; O44

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1. Introduction

Rapidly increasing tourism and quickly changing environment are important features of contemporary economies. These two variables not only are affected economic growth, but also affect economic growth. In order to properly understand interactions between economic growth, tourism and environment, this study develops an economic model in which these variables are interdependent with microeconomic foundation. Tourism has been analyzed by economists from different aspects (Sinclair and Stabler, 1997; Hazari and Sgro, 2004; Chao et al., 2009; Hazari and Lin, 2011; Zeng and Zhu, 2011; Ridderstaat et al., 2014; Gao et al., 2019; Nepal et al., 2019; Coca-Stefaniak, 2019; Escoto et al., 2019). Nevertheless, most of these economic studies of tourism are conducted within static frameworks. Dynamic issues related to tourism, for instance, economic growth, environmental change, and tourism fluctuations, cannot be properly examined within static frameworks. Dwyer et al. (2004) discuss the need for dynamic general equilibrium modeling when studying tourism and its interaction with the rest economy. Blake et al. (2006) also address the issue. Nevertheless, almost all these studies do not succeed in developing a dynamic general equilibrium framework with endogenous tourism. This study introduces tourism to growth theory with endogenous wealth and environment.

Tradeoffs between consumption and pollution have been extensively analyzed in the literature of formal economic analysis since the publication of the seminal papers by Ploude (1972) and Forster (1973). There are an increasing attention to environmental change in the literature of economic growth and development (Copeland and Taylor, 2004; Stern, 2004; Dasgupta et al., 2006; Tsurumi and Managi, 2010). We take account of environment on the basis of the literature. We integrate endogenous environment and dynamics of tourism into the neoclassical growth theory. The economic production and market aspects are based on the Uzawa two-sector growth model (Solow, 1956; Uzawa, 1961; Diamond, 1965; Stiglitz, 1967; Drugeon and Venditti, 2001; Zhang, 2005). To properly deal with tourism, we accept an analytical framework for small open economies (Zeng and Zhu, 2011). This study models behavior of households with an alternative approach proposed by Zhang in the early 1990s (Zhang, 1993). The model in this study is an extension of a growth model with tourism by Zhang (2012). The paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the model. Section 4 examines effects of changes in some parameters on the economic system over time. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. The growth model with environment and tourism

We now build a small open growth model with endogenous wealth and environment. The modelling of environmental sector is mainly based on Zhang (2015). Like in Chao et al. (2009) and Zhang (2012), the economy produces two goods: an internationally traded good (called industrial good) and a non-traded good (called services). There is an environmental sector which is financially supported by the government to maintain environment. The government income comes for taxing producers, domestic consumers and tourists. We assume that the constant population $\bar{N}$ is homogeneous. Domestic households consume both goods and services. We neglect possibility of emigration or/and immigration. We assume that foreign tourists consume only services. Except price of services foreign tourists are also affected by environment. The households hold wealth and land and receive income from wages, land rent, and interest payments of wealth. Land is only for residential and service use. Technologies of the production sectors are described by the Cobb-Douglas production functions, which are characterized of constant returns to scale. All markets are perfectly competitive, and capital and labor are completely mobile between the two sectors. Capital is perfectly mobile in international market. We assume that the economy is too small to affect the interest rate in the world market. The rate of interest r^* is fixed in international market. Let $T(t)$ stand for the work time of the representative household and $N(t)$ for the flow of labor services used at time t for production. We have $N(t)$ as follows:

$$N(t) \ = \ Z_j T(t) \, \bar{N} \tag{1}$$

where Z_j is the fixed level of human capital. We use subscript index, i, s, and e, to denote respectively the industrial, service, and environmental sectors. Capital depreciates at a constant exponential rate, δ_k, which is independent of the manner of use. Let $K_j(t)$ and $N_j(t)$ stand for the capital stock and labor force employed by sector $j, j \; i, s, e,$ at time t. We use τ_j to stand for the fixed tax rate on sector j, $0 < \tau_j < 1$, $j = i, s,$ and introduce $\bar{\tau}_j \equiv 1 - \tau_j$.

Industrial sector

The industrial sector uses capital and labor as inputs. We use $F_i(t)$ to represent the output level of sector i and specify,

$$F_i(t) = A_i \, \Gamma_i(E) \, K_i^{\alpha_i}(t) \, N_i^{\beta_i}(t), A_i, \alpha_i, \beta_i > 0, \alpha_i + \beta_i = 1 \tag{2}$$

where $\Gamma_i(E)$ is a function of the environmental quality measured by the level of pollution $E(t)$ and A_i, α_i and β_i are parameters. It is reasonable to assume that productivity is negatively related to the pollution level, i.e., $d\Gamma_i/dE \leq 0$. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The wage rate $w(t)$ is determined in domestic market. The marginal conditions are:

$$r_\delta = \alpha_i \, \bar{\Gamma}_i(t) \, k_i^{-\beta_i}(t), w(t) = \beta_i \, \bar{\Gamma}_i(t) \, k_i^{\alpha_i}(t) \tag{3}$$

Where,

$\bar{\Gamma}_i(t) \equiv A_i\, \bar{\tau}_i\, \Gamma_i(t), k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, j = i, s, e, r_\delta \equiv r^* + \delta_k.$

Service sector

The service sector uses three inputs, capital $K_s(t)$, labor force$N_s(t)$, and land $L_s(t)$. The production function of the service sector is

$$F_s(t) = A_s\, \Gamma_s(t)\, K_s^{\alpha_s}(t)\, N_s^{\beta_s}(t)\, L_s^{\gamma_s}(t), \alpha_s, \beta_s, \gamma_s > 0, \alpha_s + \beta_s + \gamma_s = 1 \quad (4)$$

where A_s, α_s, β_s, and γ_s are parameters. Let $p(t)$ and $R(t)$ stand respectively for the price of the service and the land rent. The marginal conditions for the service sector are

$$r_\delta = \alpha_s\, \bar{\Gamma}_s(t)\, p(t)\, k_s^{\alpha_s - 1}(t)\, l_s^{\gamma_s}(t), w(t) = \beta_s\, \bar{\Gamma}_s(t) p(t) k_s^{\alpha_s}(t) l_s^{\gamma_s}(t),$$

$$R(t) = \gamma_s \bar{\Gamma}_s(t) p(t) k_s^{\alpha_s}(t) l_s^{\gamma_s - 1}(t) \quad (5)$$

where
$\bar{\Gamma}_s(t) \equiv A_s\, \bar{\tau}_s\, \Gamma_s(t), l_s(t) \equiv \dfrac{L_s(t)}{N_s(t)}.$

Full employment of capital and labor

The total capital stocks employed by the country, $K(t)$, is used by the three sectors. As full employment of capital is assumed, we have

$$K_i(t) + K_s(t) + K_e(t) = K(t) \quad (6)$$

For the labor market we have

$$N_i(t) + N_s(t) + N_e(t) = N(t) \quad (7)$$

Behavior of domestic households

We use L to denote the total land available for residential and service use. Each household gets income from land ownership, wealth and wage. This study assumes that the land is equally owned by the population. Each household receive the rent income

$$\bar{r}(t) = \frac{L\, R(t)}{\bar{N}} \quad (8)$$

Consumers make decisions on lot size, consumption levels of industrial goods and services as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang (1993). We use $\bar{k}(t)$ to stand for the physical wealth owned by the representative household. The current income of the representative household is

$$y(t) = \hat{\tau}_k \, r^* \, \bar{k}(t) + \hat{\tau}_w \, h \, T(t) \, w(t) + \hat{\tau}_L \, \bar{r}(t) \tag{9}$$

where $r^* \bar{k}(t)$ is the interest payment $hT(t)w(t)$ the total wage income, and
$\hat{\tau}_m \equiv 1 - \tilde{\tau}_m, m = k, w, L,$

where $\tilde{\tau}_k$, $\tilde{\tau}_w$ and $\tilde{\tau}_L$ are respectively the fixed tax rates on the wealth (excluding land) income, wage, and land income. We call $y(t)$ the current (disposable) income in the sense that it comes from consumers' wages and current earnings from ownership of wealth. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income at any point in time is

$$\hat{y}(t) = y(t) + \bar{k}(t) \tag{10}$$

The disposable income is used for saving and consumption. At time t the consumer has the total amount of income equaling $\hat{y}(t)$ to distribute between consuming and saving. The representative household's budget constraint is

$$(1 + \tilde{\tau}_R) \, R(t) \, l(t) + (1 + \tilde{\tau}_s) p(t) \, c_s(t) + (1 + \tilde{\tau}_i) c_i(t) + s(t) = \hat{y}(t) \tag{11}$$

where $\tilde{\tau}_R$, $\tilde{\tau}_s$, and $\tilde{\tau}_i$ are respectively the consumer tax rates on housing, consumption of services, and consumption of goods. Equation (11) means that the consumption and saving exhaust the consumers' disposable income. Let $\bar{T}(t)$ stand for the leisure time. The time constraint is

$$T(t) + \bar{T}(t) = T_0 \tag{12}$$

where T_0 is the total time available for work and leisure. Substituting (12) into (11) yields:

$$\hat{\tau}_w \, h \, \bar{T}(t) \, w(t) + (1 + \tilde{\tau}_R) R(t) \, l(t) + (1 + \tilde{\tau}_s) p(t) \, c_s(t) + (1 + \tilde{\tau}_i) c_i(t) + s(t) = \bar{y}(t) \tag{13}$$

where
$$\bar{y}(t) = (1 + \hat{\tau}_k \, r^*) \bar{k}(t) + \hat{\tau}_w \, h \, T_0 \, w(t) + \hat{\tau}_L \, \bar{r}(t) \tag{14}$$

The utility function $U(t)$ is dependent on $\bar{T}(t)$, $l(t)$, $c_s(t)$, $c_i(t)$ and $s(t)$ as follows:

$$U(t) = \theta\big(E(t)\big) \, \bar{T}^{\sigma_0}(t) \, l^{\eta_0}(t) \, c_s^{\gamma_0}(t) \, c_i^{\xi_0}(t) \, s^{\lambda_0}(t), \sigma_0, \eta_0, \gamma_0, \xi_0, \lambda_0 > 0$$

in which σ_0, η_0, $\gamma_0, \delta_{qh}(> 0)$ and υ_{qe}, a_{qe} are the representative household's elasticity of utility with regard to leisure time, lot size, services, industrial goods, and saving, and $\theta(E(t))$ is the amenity which is related to the level of pollutants. We call σ_0, b_{qe} $\gamma_0, \upsilon_{qe}(F_e/2\bar{N})^{a_{qe}} \left(H_q^{\theta_q}\tilde{T}_q\right)^{b_{qe}} / H_q^{\pi_{qe}}$ and $F_e/2\bar{N}$ propensities to consume the leisure time, to use the lot size, to consume services, to consume industrial goods, and to hold wealth, respectively. Maximizing $H_q^{\theta_q} T_{qe}$. subject to the budget constraint yields

$$\bar{T}(t) = \frac{\sigma\bar{y}(t)}{w(t)}, l(t) = \frac{\eta\bar{y}(t)}{R(t)}, c_s(t) = \frac{\gamma\bar{y}(t)}{p(t)}, c_i(t) = \xi\bar{y}(t), s(t) = \lambda\bar{y}(t) \tag{15}$$

where

$$\sigma \equiv \frac{\rho\sigma_0}{\hat{\tau}_w h}, \eta \equiv \frac{\rho\eta_0}{1 \rightleftarrows +\tilde{\tau}_R}, \gamma \equiv \frac{\rho\gamma_0}{1 \rightleftarrows +\tilde{\tau}_s}, \xi \equiv \frac{\rho\xi_0}{1 \rightleftarrows +\tilde{\tau}_i}, \lambda \equiv \rho\lambda_0,$$
$$\rho \equiv \frac{1}{\sigma_0 + \eta_0 + \gamma_0 + \xi_0 + \lambda_0}$$

The change in wealth of the household follows:

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) \tag{16}$$

Demand function of foreign tourists

Let $y_f(t)$ stand for the disposable income of foreign countries. According to Schubert and Brida (2009), we assume the following iso-elastic tourism demand function

$$D_T(t) = a(t)\, y_f^{\varphi}(t)\, \theta(E(t))[(1 + \tilde{\tau}_s)p(t)]^{-\varepsilon} \tag{17}$$

where φ and $\varepsilon$are respectively the income and price elasticities of tourism demand. The variable $a(t)$ is dependent on many conditions, such as infrastructures (airports and transportation systems) and social environment (like criminal rates and traffic congestions), and cultural capital (e.g., Throsby, 1999; Beerli and Martin, 2004). The amenity is a factor of attractiveness of the small country. We assume that tourists pay the same price in consumption as domestic people. In reality, tourism industry has many special features which have important effects on pricing (e.g., Marin-Pantelescu and Tigu, 2010; Stabler, *et al*., 2010).

Full use of land

The available land is fully used for housing and service production

$$l(t)\,\bar{N} + L_s(t) = L \tag{18}$$

Demand and supply for services

The equilibrium condition for services is

$$c_s(t)\,\bar{N} + D_T(t) = F_s(t) \tag{19}$$

Dynamics of pollutants and the environment sector

We now describe dynamics of the stock of pollutants $E(t)$. We assume that pollutants are created both by production and consumption. We specify the dynamics of the stock of pollutants as follows:

$$\dot{E}(t) = \theta_i\, F_i(t) + \theta_s\, F_s(t) + \tilde{\theta}_i\, c_i(t)\,\bar{N} + \tilde{\theta}_s\, c_s(t)\,\bar{N} + \tilde{\theta}_T\, D_T(t) - Q_e(t) - \theta_0\, E(t) \tag{22}$$

in which θ_i, θ_s, $\tilde{\theta}_i$, $\tilde{\theta}_s$, and $\tilde{\theta}_T$ are positive parameters and

$$Q_e(t) = A_e\, \Gamma_e(E) K_e^{\alpha_e}(t)\, N_e^{\beta_e}(t), A_e, \alpha_e, \beta_e > 0 \tag{21}$$

where $N_e(t)$ and $K_e(t)$ are respectively the labor force and capital stocks employed by the environmental sector, A_e, α_e, and β_e are positive parameters, and $\Gamma_e(E)(\geq 0)$ is a function of $E(t)$. The term $\theta_i F_i(t)$ means that pollutants that are emitted during production processes are linearly positively proportional to the output level (Gutiérrez, 2008). The parameter, $\tilde{\theta}_i$, means that in consuming one unit of the good the quantity $\tilde{\theta}_i$ is left as waste (Prieur, 2009). The parameter θ_0 is called the rate of natural purification. The term $\theta_0 E$ measures the rate that the nature purifies environment. The term $K_e^{\alpha_e} N_e^{\beta_e}$ in Q_e means that the purification rate of environment is positively related to capital and labor inputs. The function, $\Gamma_e(E)$, implies that the purification efficiency is dependent on the stock of pollutants. For simplicity, we specify Γ_e as follows $\Gamma_e(E) = E^{b_e}$, $b_e \geq 0$. This equation means that the productivity of the environment sector is positively related to the level of pollutants. The more the environment is polluted, the more pollutants the environment sector can clear per unit of time with the same inputs (in the case of $b_e > 0$). We interpret the other terms in (20) similarly. It should be noted that in equation (2) service production and service consumption appear separately. Many services are produced simultaneously as they are consumed. For instance we may consider a case that services are only face-to-face consumption. In this case if we require $\theta_s = \theta/2, \tilde{\theta}_i = \theta/2$ and $\tilde{\theta}_s = \theta/2$, then (20) becomes:

$$\dot{E}(t) = \theta_i\, F_i(t) + \theta\, F_s(t) + \tilde{\theta}_i\, c_i(t)\,\bar{N} - Q_e(t) - \theta_0\, E(t) \tag{20}$$

We see that service production and consumption appears together only in $\theta F_s(t)$. We can model similarly if some portions of services are produced simultaneously as they are consumed. With regards to industrial goods, since production processes and consumption processes are separately carried out, they appear separately in the pollution accumulation equation.

Following Zhang (2015), we now determine how the government determines the number of labor force and the level of capital employed for purifying pollution. In the literature of environmental economics, for instance, Ono (2003) introduces tax on the producer and uses the tax income for environmental improvement in the traditional neoclassical growth theory. Our study takes account of tax not only on producers, but also on consumers. The government's tax incomes consist of the incomes on the production sectors, consumption, wage income and wealth income. Hence, the government's income is given by

$$Y_e(t) = \tau_i\, F_i(t) + \tau_s\, p(t)\, F_s(t) + \tilde{\tau}_s\, p(t)\, D_T(t) + I_h(t)\, \bar{N} \tag{22}$$

where the tax income from the representative household is:

$$I_h(t) = \tilde{\tau}_R\, R(t)\, l(t) + \tilde{\tau}_i\, c_i(t) + \tilde{\tau}_s\, p(t)\, c_s(t) + \tilde{\tau}_k\, r^*\, \bar{k}(t) + \tilde{\tau}_L\, \bar{r}(t) + \tilde{\tau}_w\, h\, w(t)\, T(t).$$

For simplicity, we assume that the government's income is used up for the environmental purpose. As there are only two input factors in the environmental sector, the government budget is given by:

$$r_\delta\, K_e(t) + w(t)\, N_e(t) = Y_e(t). \tag{23}$$

We need an economic mechanism to determine how the government distributes the tax income. The government's optimal problem is defined by

$$\mathrm{Max} Q_e(t)\ \mathrm{s.t.} \tag{23}$$

The optimal solution is given by

$$r_\delta\, K_e(t) = \alpha\, Y_e(t), w(t)\, N_e(t) = \beta\, Y_e(t) \tag{24}$$

where

$$\alpha \equiv \frac{\alpha_e}{\alpha_e + \beta_e}, \beta \equiv \frac{\beta_e}{\alpha_e + \beta_e}.$$

We have thus built the dynamic growth model with endogenous wealth, environment, and tourism. The model is a synthesis of the well-known Solow-Uzawa growth models and neoclassical growth models with environment for a small open economy with tourism. We now examine the behavior of the model.

3. Simulating Motion of the National Economy

We now show that we can follow the motion of the economic system with two differential equations. The following lemma shows how we can determine the motion of all the variables in the dynamic system.

Lemma

The motion of the land rent and environment is determine by the following two differential equations

$$\dot{R}(t) = \Omega_R\big(R(t), E(t)\big),$$

$$\dot{E}(t) = \Omega_E\big(R(t), E(t)\big)$$

where Ω_R and Ω_E are functions of $R(t)$ and $E(t)$ determined in the appendix. By the following procedure we can determine all the variables as functions of $R(t)$ and $E(t)$: $k_i(t)$ and $w(t)$ by (A1) $\rightarrow$ $k_s(t)$ by (A2) $\rightarrow$ $k_e(t)$ by (A3) $\rightarrow$ $l_s(t)$ by (A5) $\rightarrow$ $p(t)$ by (A10) $\rightarrow$ $D_T(t)$ by (A11) $\rightarrow$ $N_s(t)$ by (A15) $\rightarrow$ $\bar{k}(t)$ by (A16) $\rightarrow$ $\bar{y}(t)$ by (A7) $\rightarrow$ $\bar{T}(t)$, $l(t)$, $c_s(t)$, $c_i(t)$, $s(t)$ by (15) $\rightarrow$ $T(t) = T_0 - \bar{T}(t)$ $\rightarrow$ $N(t)$ by (1) $\rightarrow$ $\bar{r}(t)$ by (8) $\rightarrow$ $K(t)$ by (A22) $\rightarrow$ $N_i(t)$ and $N_e(t)$ by (A19) $\rightarrow$ $K_m(t) = k_m(t)N_m(t)$, $m = i, s, e$ $\rightarrow L_s(t) = l_s(t)N_s(t)$ $\rightarrow F_i(t)$ by (2) $\rightarrow$ $F_s(t)$ by (4) $\rightarrow$ $Q_e(t)$ by (21) $\rightarrow Y_e(t)$ by (A20).

The lemma implies that for a given rate of interest in the global market, the economic system at any point in time can be uniquely described as functions of the land rent and environment. Hence, if we know the motion of the land rent and environment, we can determine the motion of the whole system. It should be noted that Turnovsky (1996) proposes a small open growth model in which the equilibrium growth rates of domestic capital and consumption are determined independently. It is also concluded also in some other models of small open economies with perfect capital mobility and perfect substitutability between home capital and foreign bonds that growth rate of domestic capital is determined by production conditions (see, for instance, Zeira, 1987). In our model, the total output levels, the capital stocks employed by the economy, and economic production structure are not only determined by the production conditions and the international rate of interest, but also by tastes. Moreover, consumption is not only determined by preferences but also related to the rate of interest and the production conditions. As the expressions of the analytical result are tedious, we simulate the model. We specify:

$$\Gamma_i(t) = E^{-b_i}(t), \Gamma_s(t) = E^{-b_s}(t), \Gamma_e(t) = E^{b_e}, \theta(t) = E^{-b_0}.$$

We specify parameter values as follows

$$r^* = 0.04, \delta_k = 0.05, \bar{N} = 20, T_0 = 24, L = 8, h = 2, A_i = 1.1, A_s = 1.4, A_e = 0.5,$$
$$\alpha_i = 0.33, \alpha_s = 0.25, \beta_s = 0.65, \ \alpha_e = 0.3, \beta_e = 0.7, \lambda_0 = 0.7, \xi_0 = 0.15, \gamma_0 = 0.06,$$

$\eta_0 = 0.06, \sigma_0 = 0.2, a = 1, y_f = 4, \varphi = 1.5, \varepsilon = 1.6, \tau_m = 0.01, m = i, m, c, k, w,$
$\tilde{\tau}_m = 0.01, m = k, w, L, R, s, i, \theta_i = 0.1, \theta_s = 0.05, \tilde{\theta}_i = 0.05, \tilde{\theta}_s = 0.01, \tilde{\theta}_T = 0.05,$
$\theta_0 = 0.05, b_i = 0.15, b_s = 0.15, b_e = 0.1, b_0 = 0.05.\ (26)$

The rate of interest is fixed at 4 per cent and the population is 20. We choose the value of parameters in the Cobb-Douglas production approximately equal to 0.3. Some empirical studies show that income elasticity of tourism demand is well above unity (Syriopoulos, 1995; Lanza *et al.*, 2003). According to Lanza *et al.* (2003), the price elasticity is in the range between 1.03 and 1.82, and income elasticities are in the range between 1.75 and 7.36 (Gaŕin-Mũnos, 2007). We specify the initial conditions as follows:

$E(0) = 270, R(0) = 7.5.$

It should be noted that chosen initial conditions are not important. As the equilibrium point is stable as shown below, the system will approach to the same equilibrium point from different initial conditions. We plot the motion of the dynamic system in Figure 1. As their initial values are lower than their long-term equilibrium values, the land rent and the level of pollutants are increased over time. In association with rising land rent, the price of services is enhanced. Rising price and worsened environment reduce tourist demand. In association with falling wage, the leisure time is increased. The total labor supply, total capital employed by the economy, and the GDP fall over time. The national wealth rises. The output level of the industrial sector is reduced and that of the service sector is increased. The labor and capital inputs of the service sector are increased and the labor and capital inputs of the industrial sector are reduced. The output and two inputs of the environment sector are reduced. The household consumes more the two goods, owns more wealth, and has larger lot size.

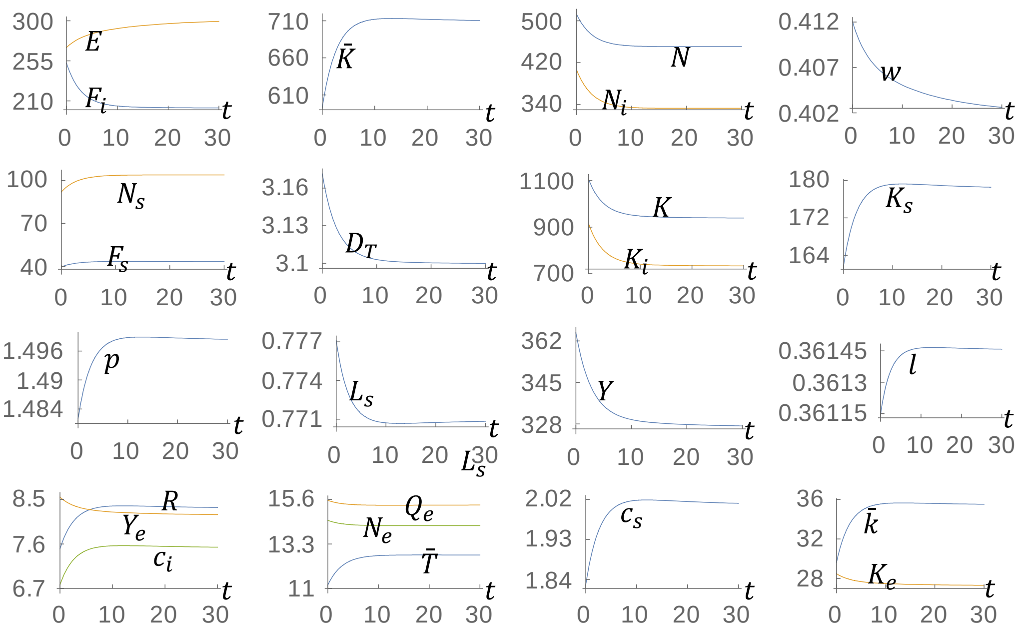

Figure 1. The Motion of the National Economy

Figure 1 shows the motion of the variables over time. From the figure we observe that all the variables of the economic system tend to become stationary in the long term. This implies that the system approaches an equilibrium point. It is straightforward to identify the following values of the equilibrium point

$p = 1.498, R = 8.32, w = 0.402, Y = 326.7, Y_e = 8.18, E = 301.8, K = 937.8, \bar{K} = 708.8, N = 451, DT = 3.1, N_i = 333, N_s = 103.8, N_e = 14.25, K_i = 732.3, K_s = 178.2, K_e = 27.27, L_s = 0.77, F_i = 201.7, F_s = 43.25, Q_e = 15.32, \bar{T} = 12.73,$

$c_i = 7.52, c_s = 2.01, l = 0.36, \bar{k} = 35.44.$

We also calculate the eigenvalues as follows: $\{-0.354, -0.071\}$. This confirms that the unique equilibrium point is stable. This result is important as the stability guarantees the validity of comparative dynamic analysis in the next section.

4. Comparative Dynamic Analysis

The previous section demonstrates the dynamics of the economy and shows the existence of a unique stable equilibrium point. This section examines how changes in some parameters affect the national economy over time. As we have shown how to simulate the motion of the system, it is straightforward to make comparative dynamic analysis. We introduce a variable $\bar{\Delta}x(t)$ to stand for the change rate of the variable $x(t)$, in percentage due to changes in the parameter value.

A rise in the rate of interest in the global market

For the small open economic system, global economic conditions such as rate of interest (which determines international flows of capital) and foreign income and preference (which determine tourist demand) are significant. It is important to study how changes in global economic conditions affect the national economy. First, we allow the rate of interest to be changed as follows: $r^* = 0.04 \Rightarrow 0.045$. The changes of the variables are plotted in Figure 2. As the cost of capital is increased, the wage rate falls. The land rent and price of services are lowered. The rise in the cost of capital causes the three sectors to use less capital. The capital stock employed by the national economy is thus reduced. The wealth owned by the country as well as the household are increased initially and are reduced in the long term. The falling in wage rate is in tandem with rising in the leisure time. The total labor supply is decreased. The labor force of the service and environment sectors are enhanced and the labor force of the industrial sector is reduced. The government has less tax income. The output levels of the three sectors are reduced. In the long term the household consumes the two products less and has less wealth.

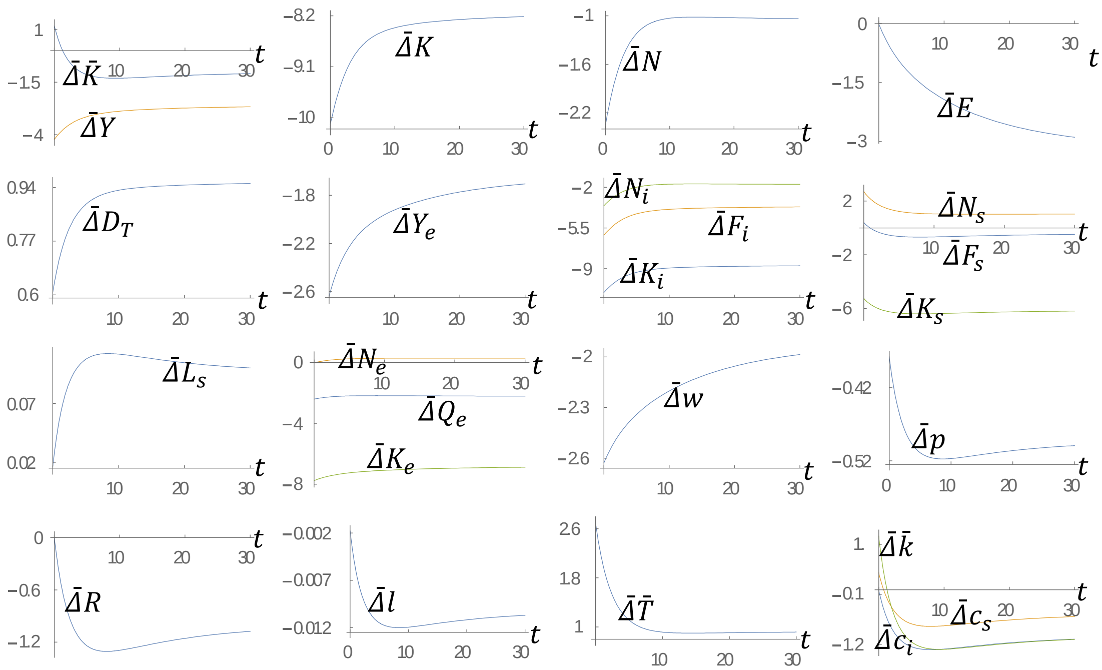

Figure 2. A Rise in the Rate of Interest

A rise in the tax rate on the service sector

We now examine what will happen to the economic system when the government increases its tax rate on the service sector as the following way: $\tau_s = 0.01 \Rightarrow 0.015$. The changes of the variables are plotted in Figure 3. The output and capital and labor inputs of the service sector are reduced as the sector suffers from paying more taxes from its unit product. The government's income is increased. The government spends more on environmental improvement. The output and capital and labor inputs of the environment sector are enhanced. The environment is improved. The productivity of the industrial sector is increased in tandem with improved environment. The wage rate rises. The land rent and price of services are increased. The GDP, total capital employed by the economy and national wealth are all increased after slight falls in initial states. The leisure time rises initially and falls in the long term. The total labor supply changes in the opposite direction of the leisure time. The improved environment attracts more tourists but increased price reduces foreign visitors. The net impact is that less foreigners come to the country. The lot size, wealth and level of industrial goods are slightly augmented. The consumption level of services falls slightly.

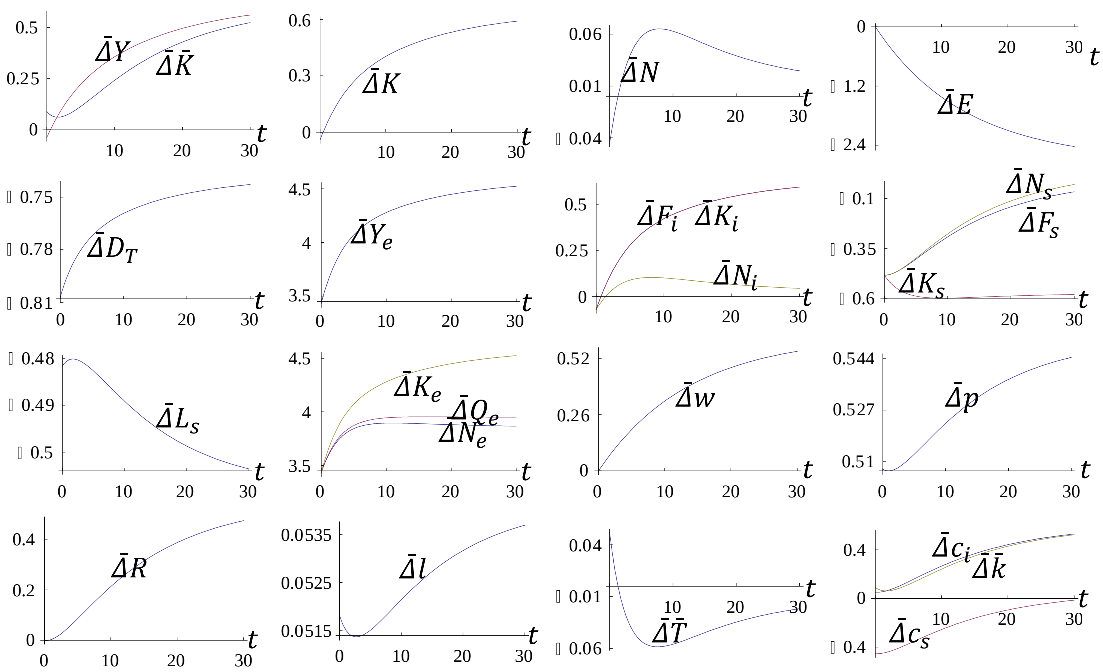

Figure 3. A Rise in the Tax Rate on the Service Sector

A rise in the tax rate on consumption of services

We just examined the effects that the government increases its tax rate on the service sector on the economic system. Rather than on the service sector, the government may change its tax rate on consumption of services. We now examine what will happen to the economic system when the government increases its tax rate on consumption of services as follows: $\bar{\tau}_s = 0.01 \Rightarrow 0.015$. The results are plotted in Figure 4. By comparing Figure 3 and Figure 4, we see that the effects in changing τ_s are almost the same as those in changing $\bar{\tau}_s$.

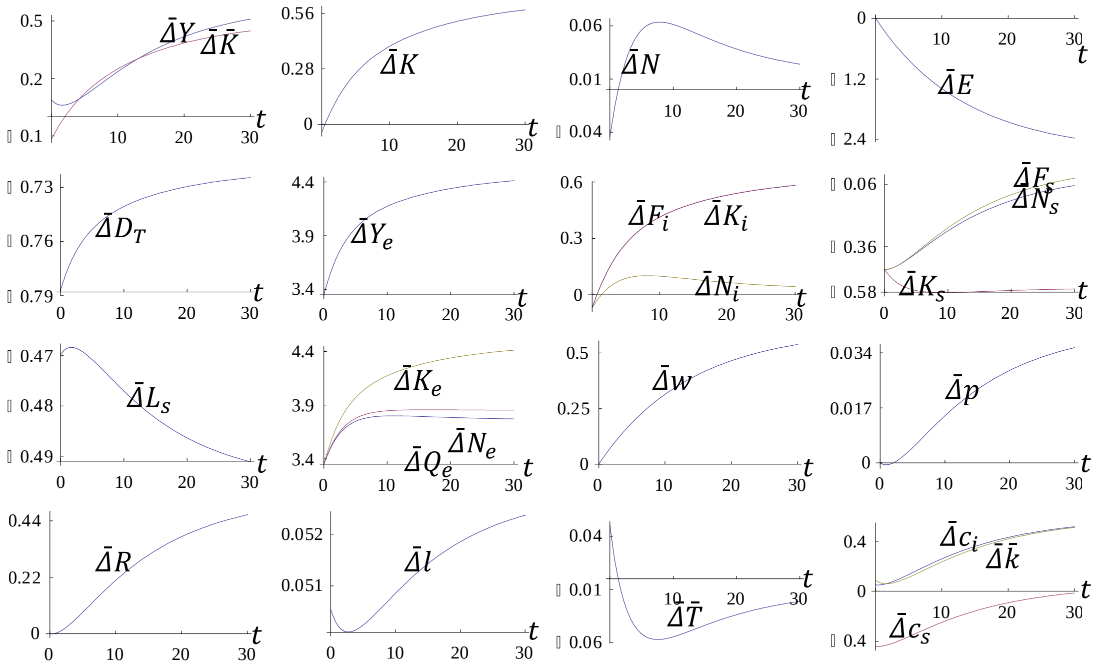

Figure 4. A Rise in the Tax Rate on Consumption of Services

Global economic conditions being improved

We now study what will happen to the national economy when the foreign income is changed as follows: $y_f = 4 \Rightarrow 4.1$. Improved global economic conditions make more foreigners to tour the economy. The price of services and output of the service sector are increased slightly. The output and capital and labor inputs of the service sector are enhanced. The output and capital and labor inputs of the industrial sector are lowered. The national resources are partly absorbed by the service sector. It should be also noted that the study by Chao *et al.* (2006) shows that an expansion of tourism can result in capital decumulation in a two-sector dynamic model with a capital-generating externality. As far as the long term impact is concerned our simulation demonstrates the same conclusion with endogenous environment. The total capital used by the country is enhanced. The leisure time is reduced slightly and is increased in the long term. The total labor supply is augmented slightly and is lowered in the long term. The service sector employs more land and the lot size is reduced. The wage and land rent are increased. The government gets more income. The environment sector's output and capital and labor inputs are increased. The environment is improved. The consumption levels of goods and services, wealth level are all reduced initially but increased in the long time. We see that an improvement in the global income reduces the living conditions and wealth of the domestic household in short term but improves these variables in the long term. It should be noted that empirical studies in the literature demonstrate an opposite relationship between a tourism boom and economic development (see, for instance, Balaguer and Cantavella-Jorda, 2002; Dritsakis, 2004; Durbarry, 2004; Oh, 2005; Kim *et al*. 2006). According to Harari and Sgro (1995) an increase in the international tourism leads to a positive effect on long-run economic growth. Our result shows that this conclusion is true as the GDP is increased. Similarly their model shows that for a small open economy the growth in tourist consumption of services increases the welfare. Our simulation demonstrates that this conclusion is valid in the long term, but not necessarily true in the short term. We get this "new insight" because our model explicitly shows transitional processes of the economic dynamics.

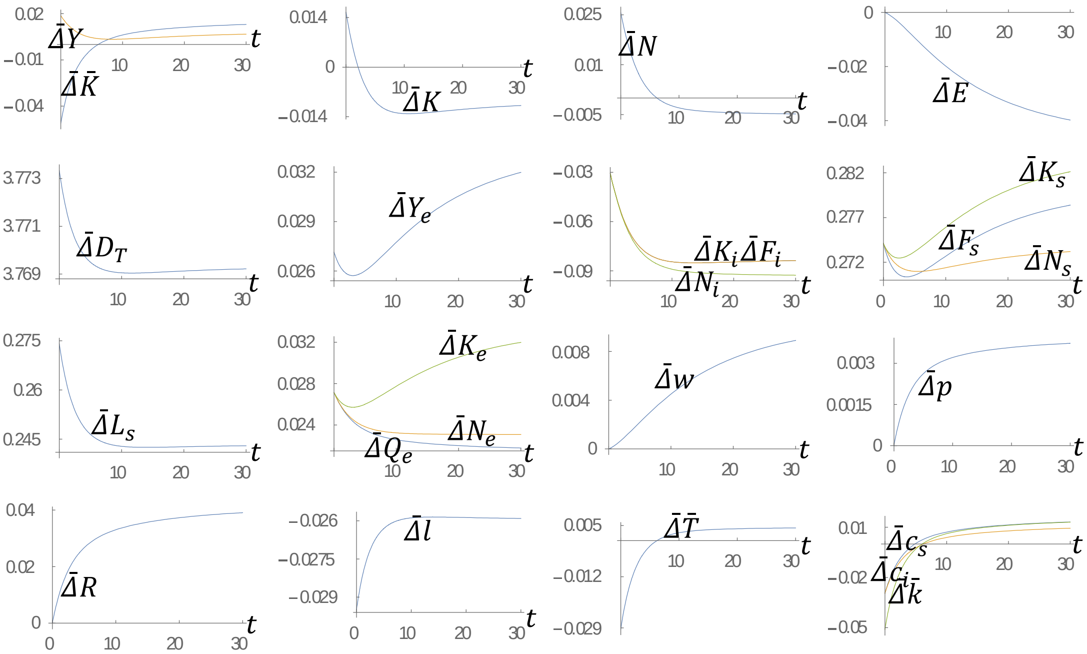

Figure 5. Global Economic Conditions Being Improved

The price elasticity of tourism being enhanced

We now examine the impact of the following rise in the price elasticity of tourism: $\varepsilon = 1.6 \Rightarrow 1.7$. The changes in the variables are plotted in Figure 6. A rise in the price elasticity reduces the tourist demand and the price of services. The output of the service sector is reduced as the demand falls. The time distribution and total labor supply are slightly affected. The total capital employed by the economy is reduced initially and increased in the long term. The GDP is reduced. The output and capital and labor inputs of the industrial sector are increased. The lot size rises, while the land input of the service sector is reduced. The consumption levels of industrial goods and services and wealth level are reduced in the long term. The government's income is reduced. The output and capital and labor inputs of the environment sector are reduced. The reduced effort in protecting environment implies the deterioration of environment.

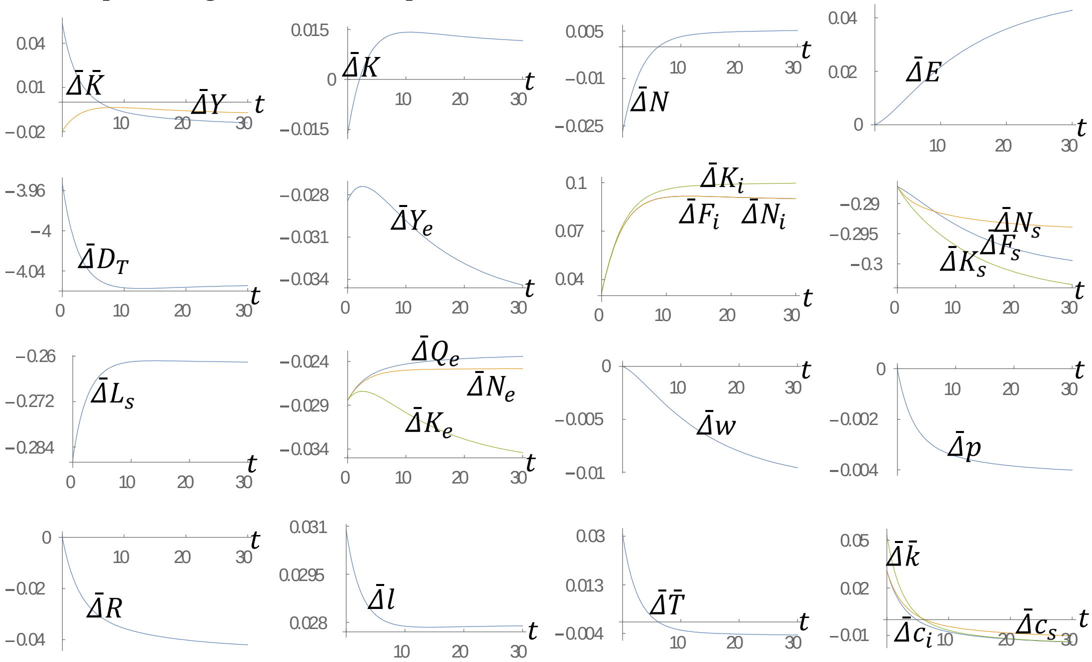

Figure 6. The Price Elasticity of Tourism Being Enhanced

A rise in the total productivity of the service sector

We now examine the impact of the following change in the total productivity of the service sector: $A_s = 1.4 \Rightarrow 1.5$. We plot the effects on the variables in Figure 7. The increased productivity of the service sector raises the output and reduces the price of services. More foreign tourists are attracted to the country. The output level and two inputs of the service sector are augmented. The output level and two inputs of the industrial sector are lowered. The GDP falls over time. The land rent is reduced. The lot size is reduced. The land input employed by the service sector is increased. The work time is increased initially and is reduced in the long term. The country uses less capital stocks and owns less wealth. The household consumes more services and owns slightly less wealth and consumes less industrial goods. The government gets more income initially and less in the long term. The output and labor input of the environment sector are increased and the capital input is lowered. The environment is deteriorated. This occurs due to the net effects of different changes. The industrial sector and consumption of industrial goods are slightly affected, which improve environment but slightly. The service production and consumption and tourism are largely affected, which deteriorate environment greatly.

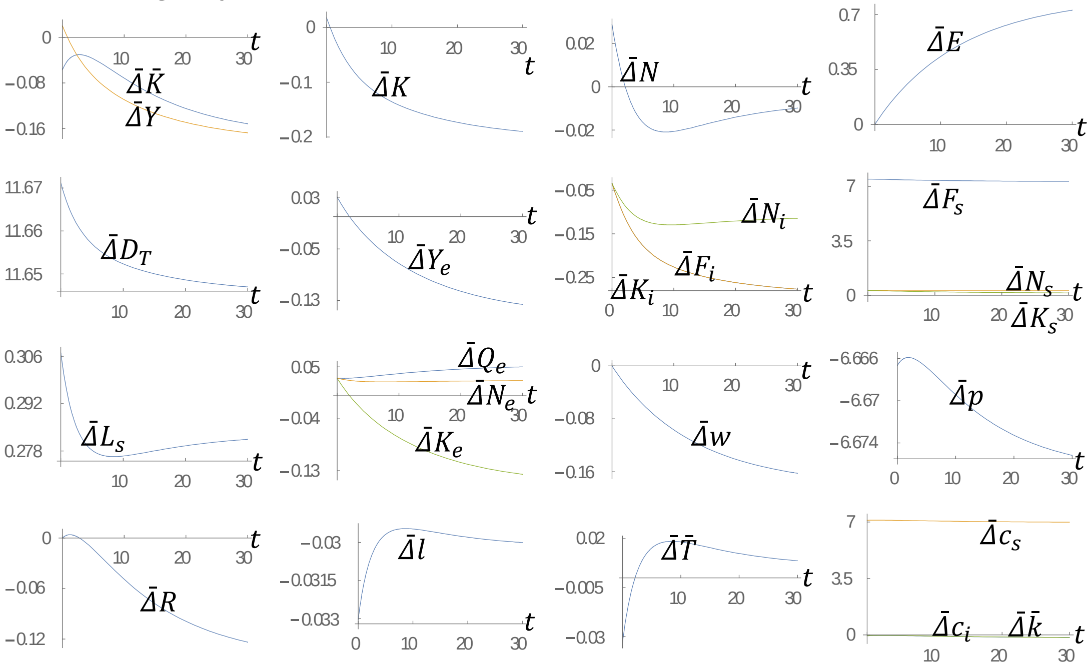

Figure 7. A Rise in the Total Productivity of the Service Sector

An improvement in human capital

We now examine what will happen to the national economy when the workers apply their human capital more effectively in the following way: $h = 2 \Rightarrow 2.2$. The changes in the variables are plotted in Figure 8. The wage rate is reduced in tandem with increasing labor supply. The leisure time is reduced initially and is slightly affected in the long term. The GDP is increased. The price of services and the land rent are increased. The increased price and worsening environment reduces foreign tourists. Environmental deterioration is the net result of positive and negative effects. The negative effects on environment are due to increased service production and consumption and industrial output. The positive effects are due to improved human capital (which increases the productivity of the environment sector) and reduced tourism. The lot size is increased and the land use of the service sector is reduced. The country uses more capital stocks. The national wealth is reduced initially and increased in the long term. The government's tax income is augmented. The household consumes more goods and services. Each sector increases their inputs and output level.

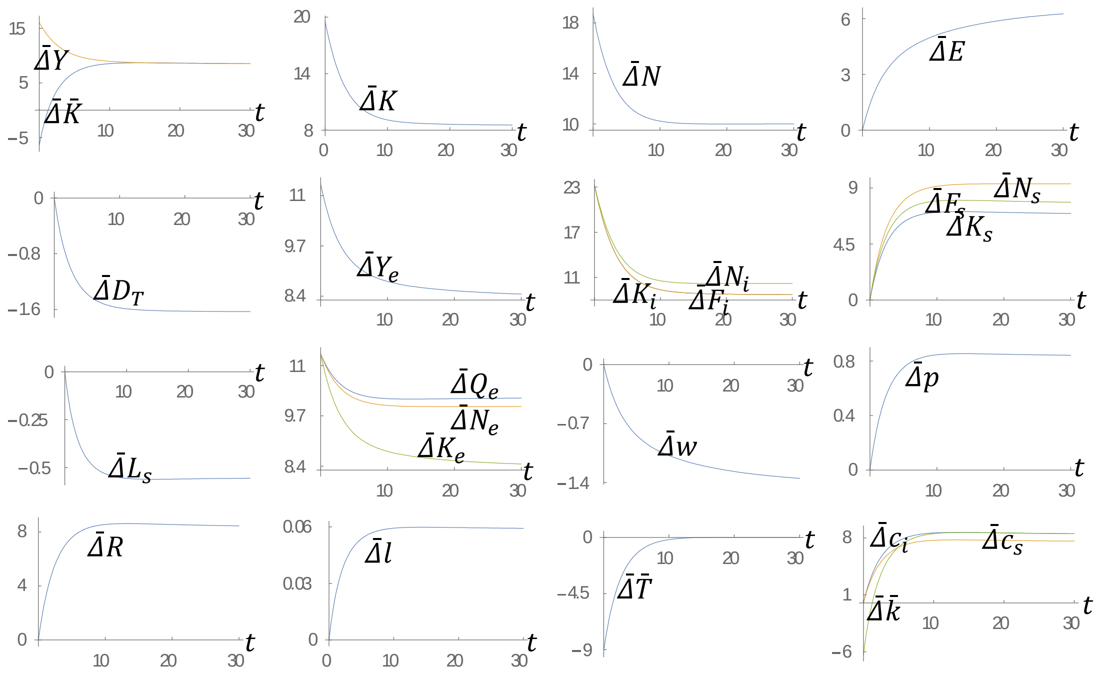

Figure 8. An Improvement in Human Capital

The household's propensity to save being enhanced

We now allow the propensity to save to be increased as follows: $\lambda_0 = 0.7 \Rightarrow 0.72$. The changes in the variables are plotted in Figure 9. The household's wealth and national wealth are increased. The leisure time is increased in tandem with rising wage rate. The total labor supply is reduced in association with falling labor time. The price of services and the land rent are increased. The household uses more lot sizes, consumes more services and industrial goods, and owns more wealth. The economy employs more capital less capital. The GDP is reduced slightly. The net impact of rising price and improved environment attracts less tourists. The two inputs and output level of the service sector are increased and the two inputs and output level of the industrial sector are lowered. The government's tax income is augmented. The two inputs and output level of the environment sector are increased.

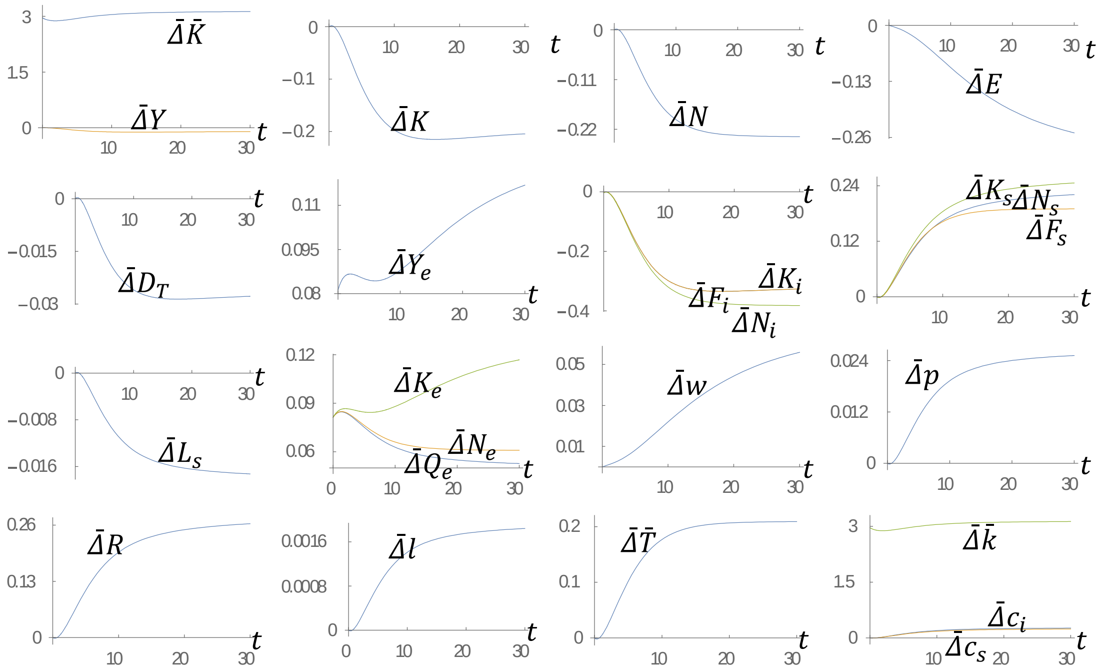

Figure 9. The Household's Propensity to Save Being Enhanced

5. Conclusions

This paper built a growth model of a small open economy with tourism and endogenous wealth and environment in a perfectly competitive economy. The national economy consists of three – service, industrial and environment – sectors. Following the traditional literature of small open economies, we assume that the rate of interest is fixed in international market. The production side is the same as in the neoclassical growth theory. We used a utility function, which determines saving, consumption and time distribution. We simulated the motion of the model and examined effects of changes in the rate of interest, the price elasticity of tourism, the government's tax rates on the service sector and consumption of services, the total productivity of the service sector, the propensity to save, and the impact of tourism on human capital accumulation. The comparative dynamic analysis provides some important insights. For instance, as global economic conditions are improved, the country will attract more tourists; the price of services and output of the service sector are increased; the output and capital and labor inputs of the service sector are enhanced; the output and capital and labor inputs of the industrial sector are lowered; the total capital used by the country is enhanced; the leisure time is reduced slightly and is increased in the long term; the total labor supply is augmented slightly and is lowered in the long term; the service sector employs more land and the lot size is reduced; The wage and land rent are increased; the government gets more income; the environment sector's output and capital and labor inputs are increased; the environment is improved; the consumption levels of goods and services, wealth level are all reduced initially but increased in the long time. It should be remarked that the model can be extended and generalized in different directions. For instance, it is important to study the economic dynamics when utility and production functions are taken on other functional forms. We do not consider possibilities that domestic households travel to other countries. This implies that it is important to deal with economies as an integrated whole (Clive *et al*., 2014). Monetary issues such as exchange rates and inflation policies are important for understanding trade issues.

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Appendix: Proving the Lemma

From (3) we have

$$k_i(E) = \left(\frac{\alpha_i \bar{\Gamma}_i}{r_\delta}\right)^{1/\beta_i}, w(E) = \beta_i \bar{\Gamma}_i k_i^{\alpha_i}, \text{(A1)}$$

where we omit time variable in expressions. Hence, we can treat k_i and w as functions of E. From (5) we solve:

$$k_s(E) = \frac{\alpha_s\, w}{\beta_s\, r_\delta}. \text{(A2)}$$

Hence, we treat k_s as a function of E. From (24) we have

$$k_e(E) = \frac{\alpha\, w}{\beta\, r_\delta}. \text{(A3)}$$

From (6) and (7) and the definitions of k_j

$$k_i N_i + k_s N_s + k_e N_e = K, N_i + N_s + N_e = N. \text{(A4)}$$

From (5), we solve

$$l_s = \frac{w\, \gamma_s}{\beta_s\, R}. \text{(A5)}$$

Insert (A5) in (18)

$$l\bar{N} + \frac{w\, \gamma_s\, N_s}{\beta_s\, R} = L. \text{(A6)}$$

From the definition of $\bar{y}$, we have

$$\bar{y} = \tau^* \bar{k} + \hat{\tau}_w\, h\, T_0\, w + \frac{\hat{\tau}_L\, L\, R}{\bar{N}}, \text{(A7)}$$

where $\tau^* \equiv 1 + \hat{\tau}_k r^*$. From (A7) and $l = \eta \bar{y}/R$ in (15)

$$\frac{l}{\eta} = \frac{\tau^*\, \bar{k} + \hat{\tau}_w\ hT_0\, w}{R} + \frac{\hat{\tau}_L\, L}{\bar{N}}. \text{(A8)}$$

Insert (A8) in (A6)

$$\bar{k} + \hat{\tau}_0 N_s = \hat{\tau}_1, \text{(A9)}$$

where

$$\hat{\tau}_1(R,E) \equiv \left(\frac{1}{\eta} - \hat{\tau}_L\right)\frac{L\,R}{\tau^*\,\bar{N}} - \frac{\hat{\tau}_w\,h\,T_0\,w}{\tau^*}, \hat{\tau}_0(E) \equiv \frac{w\,\gamma_s}{\tau^*\,\beta_s\,\eta\,\bar{N}}.$$

From (5)

$$p(R,E) = \frac{R}{\gamma_s\,\bar{\Gamma}_s\,k_s^{\alpha_s}\,l_s^{\gamma_s - 1}}. \text{(A10)}$$

By (17) and (A10)

$$D_T(R,E) = a\,y_f^{\varphi}\theta[(1+\tilde{\tau}_s)p]^{-\varepsilon}. \text{(A11)}$$

From $r_\delta = \alpha_s \bar{\tau}_s p F_s / K_s$ and (19) we have:

$$c_s\bar{N} + D_T = \frac{r_\delta\,K_s}{\alpha_s\,\bar{\tau}_s\,p}. \text{(A12)}$$

Insert $c_s = \gamma\bar{y}/p$ in (A12)

$$\gamma\bar{y}\bar{N} + pD_T = \frac{r_\delta K_s}{\bar{\tau}_s\alpha_s}. \text{(A13)}$$

Insert (A7) in (A13)

$$\tau^*\,\bar{N}\,\bar{k} + \hat{\tau}_w\,h\,T_0\,\bar{N}\,w + \hat{\tau}_L\,L\,R + \frac{p\,D_T}{\gamma} = \frac{r_\delta\,K_s}{\gamma\,\bar{\tau}_s\,\alpha_s}. \text{(A14)}$$

Insert (A9) in (A14)

$$N_s(R,E) = \left(\tau^*\,\bar{N}\,\hat{\tau}_1 + \hat{\tau}_w\,h\,T_0\,\bar{N}\,w + \hat{\tau}_L\,L\,R + \frac{pD_T}{\gamma}\right)\left(\frac{r_\delta\,k_s}{\gamma\,\bar{\tau}_s\,\alpha_s} + \tau^*\,\bar{N}\,\hat{\tau}_0\right)^{-1}, \text{(A15)}$$

where we also use $K_s = k_s N_s$. Hence we have $K_s(R,E) = k_s N_s$. By (A9) and (A15)

$$\bar{k} = \Lambda(R,E) \equiv \hat{\tau}_1 - \hat{\tau}_0 N_s. \text{(A16)}$$

By (A7) we have $\bar{y}(R,E)$. From the results so far, (15), (8), the time constraint and (1), we solve $\bar{T}, l, c_s, c_i, s, \bar{r}, T, N$ as functions of R and E. From its definition and the results so far, we have

$$I_h(R,E) = \tilde{\tau}_R\, R\, l + \tilde{\tau}_i\, c_i + \tilde{\tau}_s\, p\, c_s + \tilde{\tau}_k\, r^*\, \bar{k} + \tilde{\tau}_L\, \bar{r} + \tilde{\tau}_w\, h\, w\, T. \text{(A17)}$$

By (A4)

$$k_i\, N_i + k_e\, N_e = K - k_s N_s, N_i + N_e = N - N_s. \text{(A18)}$$

Solve (A18)

$N_i = f_i - k_0\, K, N_e = k_0\, K + f_e$, (A19)

where

$$k_0(R,E) \equiv \frac{1}{k_e - k_i}, f_i(R,E) \equiv k_0\, k_e\, N - (k_e - k_s) k_0\, N_s,$$
$$f_e(R,E) \equiv -k_0\, k_i\, N + (k_i - k_s) k_0\, N_s.$$

From (22), (2) and (4), we have

$$Y_e \;=\; \tau_i\, A_i\, \Gamma_i\, k_i^{\alpha_i}\, N_i + \tau_s\, p\, A_s\, \Gamma_s\, k_s^{\alpha_s}\, l_s^{\gamma_s}\, N_s + \tilde{\tau}_s\, p\, D_T + I_h\, \bar{N}. \text{(A20)}$$

Insert (24) in (A20)

$$\frac{w\, N_e}{\beta} = \tau_i\, A_i\, \Gamma_i\, k_i^{\alpha_i}\, N_i + \tau_s\, p\, A_s\, \Gamma_s\, k_s^{\alpha_s}\, l_s^{\gamma_s}\, N_s + \tilde{\tau}_s\, p\, D_T + I_h\, \bar{N}. \text{(A21)}$$

Insert (A19) in (A21)

$$K(R,E) = \Big(\tau_i\, A_i\, \Gamma_i\, k_i^{\alpha_i}\, f_i - \frac{f_e\, w}{\beta} + \tau_s\, p\, A_s\, \Gamma_s\, k_s^{\alpha_s}\, l_s^{\gamma_s}\, N_s + \tilde{\tau}_s\, p\, D_T$$
$$+ I_h\, \bar{N}\Big) f, \text{(A22)}$$

where

$$f \equiv \left(\frac{w}{\beta} + \tau_i\, A_i\, \Gamma_i\, k_i^{\alpha_i}\right)^{-1} \frac{1}{k_0}.$$

In sum, by the following procedure we can determine all the variables as functions of R and E: k_i and w by (A1) $\rightarrow$ k_s by (A2) $\rightarrow$ k_e by (A3) $\rightarrow$ l_s by (A5) $\rightarrow p$ by (A10) $\rightarrow D_T$ by (A11) $\rightarrow N_s$ by (A15) $\rightarrow \bar{k}$ by (A16) $\rightarrow \bar{y}$ by (A7) $\rightarrow \bar{T}, l, c_s, c_i, s$ by (15) $\rightarrow T = T_0 - \bar{T}$ $\rightarrow N$ by (1) $\rightarrow \bar{r}$ by (8) $\rightarrow K$ by (A22) $\rightarrow N_i$ and N_e by (A19) $\rightarrow K_m = k_m N_m$, $m = i, s, e \rightarrow L_s = l_s N_s \rightarrow F_i$ by (2) $\rightarrow F_s$ by (4) $\rightarrow Q_e$ by (21) $\rightarrow Y_e$ by (A20). From this procedure, (16) and (20), we have

$$\dot{\bar{k}} = \Omega_0(R, E) \equiv s - \bar{k}, \text{(A23)}$$
$$\dot{E} = \Omega_E(R, E) \equiv \theta_i\, F_i + \theta_s\, F_s + \tilde{\theta}_i\, c_i\, \bar{N} + \tilde{\theta}_s\, c_s\, \bar{N} + \tilde{\theta}_T\, D_T - Q_e - \theta_0\, E. \text{(A24)}$$

Taking derivatives of (A16) with respect to time yield

$$\dot{\bar{k}} = \frac{\partial \Lambda}{\partial R}\, \dot{R} + \Omega_E\, \frac{\partial \Lambda}{\partial E}. \text{(A25)}$$

We do not provide the expressions in the above equation because it is tedious. Equal the right-hand sides of (A23) and (A25)

$$\dot{R} = \Omega_R(R, E) \equiv \left(\Omega_0 - \Omega_E\, \frac{\partial \Lambda}{\partial E}\right)\left(\frac{\partial \Lambda}{\partial R}\right)^{-1}. \text{(A26)}$$

We thus proved the lemma.