

COMPARISON OF VARIOUS SKY MODEL FOR DAYLIGHT AVAILABILITY INSIDE THE CLASSROOM WITH BILATERAL OPENING TYPOLOGY IN THE TROPICS

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ABSTRACT

This study compares daylighting performance under four sky models of a classroom in tropical climates to understand the differences in illuminance and uniformity values. This research is significant as it can inform the relevance of the widely used static metric, such as the daylight factor, for daylight performance evaluation in tropical climates in comparison with the climate-based sky model which is utilized for dynamic metric calculation. Computational simulation was employed to achieve the objective. Grasshopper-Rhinoceros was utilized for the classroom model, while Radiance was employed for sky modelling and daylight simulation. The results indicated that static sky models exhibited greater discrepancies in their average illuminance and uniformity values compared to climate-based or dynamic sky models. The pervasive utilization of static metrics, such as the daylight factor, for evaluating daylighting performance within a space may necessitate reconsideration in tropical climates, given the higher error rates observed in this study for a classroom with bilateral opening design.

Keywords: Daylighting, Sky Model, Classroom, Bilateral Opening, Tropics

1. Introduction

The role of daylight modeling in the evaluation of building performance is of significant, as it informs decisions relating to daylight availability and visual comfort. Sky models are crucial for forecasting the daylight availability within interior spaces, as they represent the light source. The evolution of sky models has progressed from the initial development of simple mathematical models, such as the uniform, Moon & Spencer, and Kittler models (Kittler, 1967; Moon & Spencer, 1947; Yamauti, 1924), to more sophisticated ones, including the Tregenza and Perez models (Perez et al., 1993; Tregenza & Waters, 1983). In the tropics, for instance in Indonesia, the country standard (Badan Standardisasi Nasional (BSN), 2001) for daylight evaluation is still utilizing static metric which is sky component under uniform sky model (Yamauti, 1924).

While the static metric offers a straightforward calculation approach, it is unable to account for the microclimate variability present at diverse locations. This is due to the fact that the calculation was based on a limited number of assumptions as a result of the technological constraints associated with the initial discovery of the static sky models. For example, in Indonesia, an initial study revealed significant discrepancies between the Commission Internationale de l'Eclairage (CIE) overcast sky and the uniform sky (Hakim et al., 2021a), with a maximum error of 163%. The uniform sky (Yamauti, 1924) was one sky model proposed long before other models adopted by the CIE, such as the CIE standard overcast sky (Moon & Spencer, 1947) and the CIE standard clear sky (Kittler, 1967). Meanwhile, since the adoption of the Moon and Spencer cloudy sky as the standard cloudy sky, the CIE has no longer included the uniform sky as one of the standard sky models used for DF calculations.

Further study has indicated that utilization of the useful daylight illuminance (UDI) metric, with an illuminance range of 250~750 lux, represents an effective and representative dynamic metric in Indonesia (Atthaillah et al., 2022b, 2022a; Atthaillah et al., 2024a, 2024b). In instances where the average UDI in this proposed range exceeds 80%, it can be assumed that all other dynamic metric criteria are also met under the classroom with bilateral opening typology

(Atthaillah et al., 2024a, 2024b). In addition, other studies employ a dynamic metric to assess daylight availability inside a space (Bian et al., 2023; Korsavi et al., 2016; Moreno & Labarca, 2015; Samiou et al., 2022). The employment of the aforementioned dynamic metric suggests the use of more complex sky models based on the climatic data of a location, such as the Perez (Perez et al., 1993) and Tregenza sky models (Tregenza & Waters, 1983).

Despite the widespread use of various sky models, their accuracy in predicting illuminance and uniformity, particularly those with bilateral openings such as the Indonesian school classroom, remains insufficiently understood. This lack of clarity affects daylighting assessments and can lead to suboptimal design choices in terms of daylight availability and visual comfort. Therefore, this study proceeds to understand better this situation in the tropics (i.e., an Indonesian location), where it might either be appropriate to employ the static metrics since the relatively constant daylight availability throughout the year or vice versa.

The objective of this study is to comprehend the discrepancies and uncertainty in the mean illuminance and uniformity values within the classroom and their associated error values. It is of the utmost importance to comprehend the specific sky model utilization when evaluating the daylight conditions within a given space, for instance a classroom, in the tropics, as this understanding is fundamental to the formulation of design decisions (i.e., daylighting design).

2. Literature Review

The static metric, specifically the sky component (SC) and DF metrics, as outlined in the Indonesian daylighting standard (Badan Standardisasi Nasional (BSN), 2001), was selected for its straightforward calculation process (Atthaillah et al., 2022a; Atthaillah et al., 2024a). The difference between SC and DF metrics is that SC does not include the reflectance of the interior and exterior surfaces, nor the transmittance of the glass. The calculation is based only on the effective opening contribution to daylighting, while the DF calculation includes the surface reflectance (both interior and exterior surfaces) and the glass transmittance (Mardaljevic, 2021). The similarity of the metrics is that they often used a simpler sky model such as uniform and CIE standard overcast sky models.

However, since the adoption of the CIE standard overcast sky model, the CIE has excluded the uniform sky model from the metric calculation because it is an oversimplification of the real condition based on only one sky luminance value (Mangkuto, 2016; Reinhart, 2011). The CIE standard overcast sky (Moon & Spencer, 1947) models, the presence of the sun is not considered or only the sky glow is considered. In its development, a clear sky model was proposed in 1967 (Kittler, 1967). In this sky model, the presence of the sun is considered, however, without variations in climatic conditions such as the presence of clouds or rainy conditions. Therefore, this sky model is not sensitive to certain climatic conditions.

Since the development of computational tools, sky models have evolved from a simple mathematical model to a more complex sky model that represents the climatic conditions of a specific location, known as the Perez sky model (Perez et al., 1993). The sky model was developed through the integration of weather data into a continuous sky dome. This model is capable of calculating illuminance and luminance values with a high degree of accuracy for a specific location at a given hour of the day, based on the input of specific weather data. However, for annual daylight calculation, this sky model is not efficient as it requires expensive computation time. Therefore, it is not practical for annual daylight simulation.

To address the aforementioned issue, it was imperative to develop an alternative sky model that could accurately simulate annual daylight conditions with greater efficiency. Therefore, it was proposed an alternative sky model based on the Tregenza sky model. This sky model was based on the Tregenza sky subdivision which consisted of 145 disk segmentation in the sky dome, it was known as daylight coefficient model (Tregenza & Waters, 1983). Subsequently, it was proposed that the sky dome be divided into rectangular segments to address the issue of the empty space resulting from the disk construction. However, the decision was taken to retain the original design with 145 sky segmentations. This is referred to as the Tregenza sky model which is known to have multiplication factor (MF) = 1 for the sky subdivision (Reinhart & Walkenhorst, 2001).

This sky model is utilized for the annual daylight simulation due to its better time efficiency in comparison to the original Perez sky model.

With the ability to simulate annual daylight, consequently, some daylight dynamic metrics were proposed for daylight evaluation in a space. The concept of daylight autonomy (DA) was among the first metrics to be proposed for the evaluation of daylight, taking into account the impact of climate variability throughout the year (Reinhart & Walkenhorst, 2001). DA defines a single threshold for the evaluation of daylight, namely that if the daylight availability in a sensor is equal to or above 300 lux, then it is considered sufficient. Given that the deficiency of daylight due to excessive sunlight could not be justified, the concept of useful daylight illuminance (UDI) was proposed (Nabil & Mardaljevic, 2005). The sufficient threshold for the UDI metric was revised to a range of 100 lux to 3000 lux over the course of a year (Mardaljevic, 2010). UDI defines the lower and upper threshold for a sensor in a working plane within a given space, thereby enabling the insufficiency of daylight to be justified, whether due to a lack or excessive daylight, in addition to the sufficient condition. The aforementioned total availability metrics indicated that the extent to which sunlight contributed to a sensor's measurements within a given space remained inconceivable.

In 2012, the Illuminating Engineering Society of North America (IESNA) proposed an annual sunlight metric called Annual Sunlight Exposure (ASE) (IESNA, 2012). The DA concept has been further developed for the evaluation of spatial conditions, the so-called spatial daylight autonomy (sDA). The sDA and ASE later adopted as a standard annual daylight measurement metric in America (United States Green Building Council (USGBC), 2013, 2021). As previously discussed, the yearly daylight metrics employed temporal data for calculations. The temporal data depended on the evaluation hours selected on a yearly basis, with each evaluation hour at each sensor having a specific illuminance value throughout the year by default under Tregenza sky model.

Notwithstanding the aforementioned developments, certain studies continue to assess daylight under both static sky models, such as the CIE clear sky and CIE standard overcast sky models, and dynamic sky models, including the Perez and Tregenza sky models. A study was conducted to evaluate the design of a courtyard under CIE standard overcast sky conditions (Acosta et al., 2014). A square courtyard with varying heights was evaluated with height-to-width ratios between 1/3 and 5/1. Additionally, the courtyard reflectance was set at 0.3, 0.5, and 0.7. The Tregenza algorithm was employed for SC calculation. Interior reflection was calculated by subtracting the SC from the DF. The research asserted that the results of their method were accurate for DF calculation under CIE standard cloudy conditions. Similarly, a study investigating top lighting utilizing DF metric was conducted in the school setting in Slovakia (Dolníkova et al., 2020). Moreover, SC was employed for the assessment of the accuracy of simulation tools. The SC metric indicated the utilization of the static sky model (Acosta et al., 2015).

Furthermore, the accuracy of the existing static model was evaluated in regard to luminance spectral sky models. It was found that the CIE standard overcast sky model exhibited the best performance (Diakite-Kortlever & Knoop, 2021). Additionally, a novel methodology was put forth with the objective of enhancing the precision of the CIE standard sky models for a specific climate of Saudi Arabia. The Tregenza method of generating sky luminance was deemed to be overly intricate for the purpose of analysis (Alshaibani & Li, 2021). Another study sought to identify pertinent CIE standard sky models in relation to the climatic conditions in Harbin, China (Sun et al., 2021). The findings revealed that the CIE standard clear sky model was the most prevalent within the Harbin climate. A comparable endeavor to enhance the CIE standard clear sky model has been undertaken in Morocco (Mendyl et al., 2023) and Spain (Dieste-Velasco et al., 2024).

In a broader context, static sky models were also evaluated in an agricultural setting for the purpose of modeling photosynthetic active radiation (PAR) (García-Rodríguez et al., 2021). In a building context, an office with unilaterial opening was evaluated for various daylight metrics under CIE standard overcast sky condition for various façade thickness (Mangkuto et al., 2021). The variation in façade thickness had a significant impact on the observed daylight metrics, with sDA exhibiting the least influence. These results demonstrated insensitivity of sDA to daylight

condition for the specific case observed under the adapted CIE standard overcast sky model, which was representative of the typical outdoor conditions in tropical climate regions.

Furthermore, recent years have seen efforts made to evaluate the annual daylight conditions within a given space. The annual daylight has been indicated through the utilization of Perez and Tregenza sky models. Some studies have considered the one-sided opening typology for daylighting evaluation, including analyses of office and school settings (Bahdad et al., 2021; Bakmohammadi & Noorzai, 2020; Brembilla & Mardaljevic, 2019). Also, top-lighting strategies were implemented in accordance with the dynamic sky condition (Fan et al., 2021; Lou et al., 2021; Salma et al., 2023). Next, several studies assessed the two-sided opening typology for its impact on annual daylight performance. In tropical climates, where classrooms are often situated, the two-sided opening typology proved advantageous (Atthaillah et al., 2022b, 2022a; Atthaillah et al., 2024a; Effendy et al., 2023; Hakim et al., 2021b).

From previous discussions, the choice of static sky models, including the CIE standard models, remains a topic of ongoing research. This is largely attributable to the straightforwardness of these models, which facilitate the calculation of the relevant metrics. Concurrently, the debate has also given rise to the proposition of utilizing annual daylighting evaluation, which indicates the deployment of a more dynamic sky model, such as the Perez and Tregenza sky model. However, to what degree the discrepancy occurs in the daylight availability inside a space due to the change from static and dynamic sky is remain unclear particularly in the tropics. The tropical climate provides a distinctive opportunity to employ a two-sided opening typology, which is advantageous for cross-ventilation. With regard to daylighting, the bilateral opening typology is also distinctive due to the superimposition of light within a space. A space that utilizes the bilateral opening typology in a tropical setting is a classroom. In Indonesia, classrooms are a common feature throughout the country and are typically large in size. Thus, this study attempts to investigate the discrepancy and uncertainty of daylight availability under both static and dynamic sky models in a classroom setting in the tropics.

3. Research Methods

This study employed a computational method for the simulation purpose, utilizing the *Radiance* (RAD) (G. Ward & Rubinstein, 1988). The RAD is a validated simulation engine for short-term (Atthaillah et al., 2022b; Atthaillah et al., 2024a; Khidmat et al., 2022), long-term daylight simulation (Brembilla & Mardaljevic, 2019; Geisler-Moroder et al., 2017; Kharvari, 2020; Mardaljevic, 1995; McNeil & Lee, 2012; G. J. Ward et al., 2021). Moreover, approximately 52% of the daylight simulation employed RAD-based tools (Ayoub, 2019). Previously, *Daysim* was the sole daylight simulation engine capable of conducting annual daylight simulations. For annual simulations, *Daysim* considered 65 points for the sun representation, which proved problematic for accurately simulating the direct sunlight contribution inside a space (Reinhart & Walkenhorst, 2001; Subramaniam & Mistrick, 2017). Recently, RAD has been enhanced with the ability to represent the sun's position in the sky dome with greater precision through a more refined sky discretization (Subramaniam, 2017). Moreover, a RAD-based daylight simulation software has been developed to incorporate the actual sun position in the analemma (Roudsari & Pak, 2013; Subramaniam & Mistrick, 2017). It is therefore recommended that RAD be utilized in order to achieve the objectives of this research.

A classroom with bilateral opening typology (Fig. 1) was investigated under four different sky models, namely Tregenza (annual sky), Perez (continuous dynamic sky), Kittler (CIE standard clear sky with sun), and Moon & Spencer (CIE standard overcast sky) (Kittler, 1967; Moon & Spencer, 1947; Perez et al., 1993; Tregenza & Waters, 1983). The sky models were selected because they were the most relevance sky models utilized for daylight performance simulation. For instance, DF calculation for under Moon and Spencer sky model for school classroom daylight performance (Dolnikova et al., 2020), DF under clear sky condition or Kittler sky model (Ahmad et al., 2022; Nasrollahi & Shokry, 2020) and some other various studies for annual daylight performance which indicated the utilization of the dynamic sky models such as Perez (Subramaniam & Mistrick, 2017) and Tregenza sky models (Atthaillah et al., 2024a; Bahdad et al., 2020; Bian et al., 2023; Samiou et al., 2022). The bilateral openings are oriented

east and west. This orientation was selected based on the current findings that east and west-facing openings can serve as a reference orientation for evaluating daylight availability within a space in the tropics (Atthaillah et al., 2024a, 2024b). Next, the classroom dimensions were $7\text{ m} \times 8\text{ m} \times 3.5\text{ m}$, as suggested the ministry of education (Kementerian Pendidikan Nasional RI, 2011), with a 30% window-to-wall ratio (WWR) as previously suggested for tropical climate classroom (Abdelhakim, M., Lim, Y. W., & Kandar, 2019). Furthermore, the window was elevated at 1.5 m above the floor level. The window height and their distances (center-to-center) were 1.2 m and 1.5 m, respectively. Additionally, 100 sensor points were equally distributed in the classroom, elevated at 0.75 m above the floor level. The classroom was modelled using *Grasshopper* (GH) algorithms within the *Rhinoceros* (RH) interface (Robert McNeel & Associates, 2019). Subsequently, the geometry was converted into a RAD scene utilizing the *occonv* program of RAD for further daylight evaluation. Subsequently, the classroom model was assigned to the RAD material, as detailed in Table 1.

This study employed the Perez sky model as a reference point to facilitate comparison of illuminance values with those derived from alternative models. The Perez sky model is considered the most validated sky model (Mardaljevic, 2000; Perez et al., 1993) for performing dynamic daylight analyses for a specific location worldwide. However, for annual daylight evaluations, this model is not considered the most efficient due to the longer simulation times involved. For annual daylight simulations, the Tregenza sky model is utilized (Tregenza & Waters, 1983).

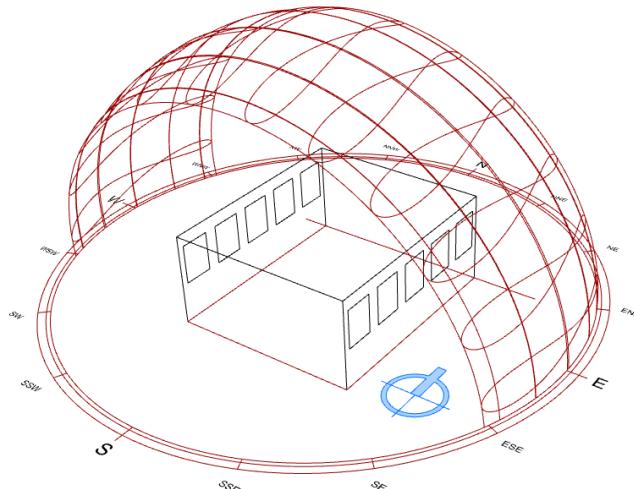


Fig. 1. Digital model of the classroom. Window facades face east and west

Table 1 - The classroom RAD material setting.

Radiance Material	Value
Wall reflectance (ρ_w) [-]	0.5
Floor reflectance (ρ_f) [-]	0.2
Ceiling reflectance (ρ_c) [-]	0.8
Ground reflectance (ρ_g) [-]	0.2
Shading reflectance (ρ_{shd}) [-]	0.3
Glass transmittance (τ) [-]	0.7

The sky model was created utilizing the *gensky* program for the Kittler and Moon & Spencer sky models. The Perez sky model was generated with the *gendaylit* program. Lastly, the Tregenza sky model was created utilizing *gendaymtx* of *Radiance*. Both the Perez and Tregenza sky models received location input from a *.wea* file for the location of Lhokseumawe, Indonesia ($5^{\circ}10'0''\text{ N}$, $97^{\circ}8'0''\text{ E}$, 2~24 m above sea level). The selected location is deemed appropriate for use as a reference point for daylight evaluation in Indonesia, based on the findings of the previous study (Atthaillah et al., 2024b). The *.wea* file was generated through the utilization of the *epw2wea* program, which accepts an input from the *.epw* file as its input. The illustration of the sky models is presented in Fig. 2a-d. The daylight was evaluated based on indoor illuminance

values at critical times of the year (equinox and solstice), specifically from 08:00-17:00. These critical days were March 21st, June 22nd, September 20th, and December 22nd, thus, this resulted in 40 evaluation hours.

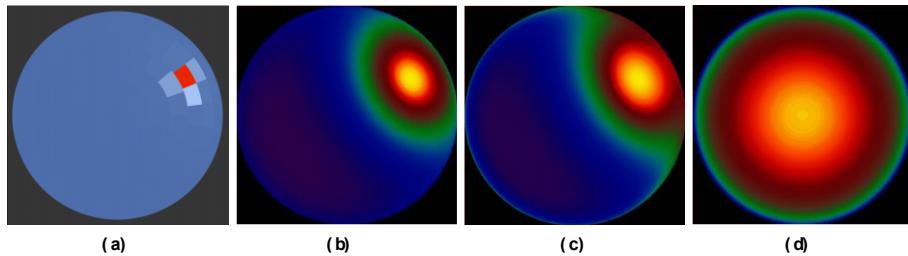


Fig. 2. Sample of sky models generated with RAD for daylight investigation in this study (a) Tregenza, (b) Perez, (c) Kittler and (d) Moon & Spencer.

For the daylight simulation inside the classroom, the *rtrace* program was employed to calculate the illuminance under the Perez, Kittler, and Moon & Spencer sky models. In contrast, the *rfluxmtx* program was utilized for the Tregenza sky model. This program established a communication channel with *rcontrib* in the process of carrying out the daylight calculation under the Tregenza sky. All the results were converted into illuminance values (lux) for 40 evaluation hours. The setting for each simulation program is shown in Table 2.

Table 2 - RAD setting for daylight simulation.

<i>Radiance</i> Program	Parameter
rtrace	-aa 0.1 -ab 6 -ad 4096 -ar 128 -as 4096 -dc 0.75 -dj 1.0 -dp 512 -ds 0.05 -dr 3 -dt 0.15
rcontrib	-aa 0.1 -ab 6 -ad 25000 -ar 128 -as 4096 -dc 0.75 -dj 1.0 -dp 512 -ds 0.05 -dr 3 -dt 0.15 -I -lr 8 -lw 4e-07 -c 1 -ss 1.0 -st 0.15

4. Data Analysis

Firstly, in order to understand the daylight distribution inside the space, uniformity (*U*) metrics are utilized. In this study, two uniformity metrics are employed as shown in equations (1) and (2).

$$U_1 = \frac{E_{min}}{E_{max}} \quad (1)$$

$$U_2 = \frac{E_{min}}{E_{mean}} \quad (2)$$

where E_{min} , E_{max} and E_{mean} are minimum, maximum, and mean illuminance values respectively. The illuminance data were obtained from the RAD simulation for four sky models.

Secondly, relative error is observed for the comparison between the illuminance (*E*) and uniformity (*U*) values from the Perez sky (reference sky model) with the rest of alternative sky models. The errors are defined as in equations (3) and (4).

$$\epsilon_E = \left| \frac{E_{ref\ sky} - E_{alternative\ sky}}{E_{ref\ sky}} \right| \times 100\% \quad (3)$$

$$\epsilon_U = \left| \frac{U_{ref\ sky} - U_{alternative\ sky}}{U_{ref\ sky}} \right| \times 100\% \quad (4)$$

where ϵ_E and ϵ_U are relative error for illuminance and uniformity values respectively. $U_{ref\ sky}$ and $U_{alternative\ sky}$ are uniformity values for reference and alternative sky models respectively. In accordance with the previously proposed 10% error tolerance threshold (Mardaljevic, 2000;

Subramaniam & Mistrick, 2017), this study proceed on the assumption that any error within this range will be considered acceptable.

Next, an uncertainty analysis was conducted for both illuminance (E) and uniformity (U), with the coefficient of variance (CV) values observed. The CV value is defined as a comparison of the standard deviation (σ) with the mean value and can be mathematically defined as in equation (5).

$$CV = \frac{\sigma_E \text{ or } \sigma_U}{E_{\text{mean}} \text{ or } U_{\text{mean}}} \quad (5)$$

Finally, Pearson correlation analysis and scatterplot were employed to evaluate the relationship between the reference and the alternative sky models. This is important to observe the correlation between the illuminance value for each sky model and the classroom. This information is crucial to understand the interpolation value between each sky model for validation purposes.

5. Results

The mean illuminance value derived from a variety of sky models for critical days of the year is presented in Fig. 3. It can be observed that the Perez and Tregenza skies have an almost comparable illuminance value throughout the critical day of the year. Both sky models indicated a different pattern of average illuminance (aE) across 40 evaluation hours. Furthermore, aE values under Kittler and Moon & Spencer sky models demonstrated a relatively predictable pattern. However, the Kittler sky model exhibited a more diverse range of aE values in comparison to the Moon & Spencer model. Only the Moon & Spencer model demonstrated a consistent pattern across all months. Furthermore, the almost comparable pattern of Perez, Tregenza and Kittler occurred in September.

Nevertheless, the average illuminance value exhibited a relatively greater discrepancy between Perez or Tregenza and the Kittler sky model. This phenomenon is presented in Table 3, where the discrepancy between the median (Perez = 4057.40 lux, Kittler = 5989.98 lux) and mean values (Perez = 4691.19 lux, Kittler = 5363.66 lux) was more pronounced when comparing Perez and Kittler sky models. Furthermore, the Moon & Spencer model exhibited an exceptionally low illuminance value (underestimate the illuminance values) in comparison to the other sky models. However, all the sky models indicated a higher degree of uncertainty over time ($CV > 0.1$) in terms of their illuminance values, with the lowest occurring under the Moon and Spencer sky model ($CV = 0.3$, Table 3).

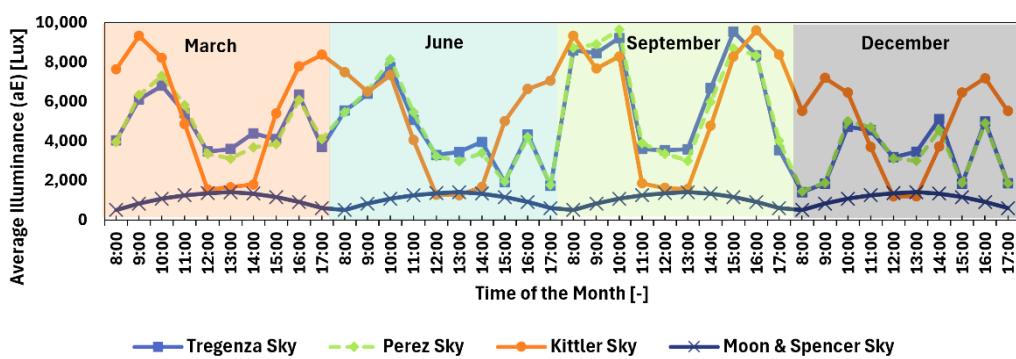


Fig. 3. Average illuminance results under various sky models across the critical day of the year.

Table 3 - Average illuminance statistics for various sky models.

Sky Model	Min.	Max.	Median	σ	Mean	CV
Perez	1448.14	9647.51	4057.40	2235.12	4691.19	0.48
Tregenza	1409.94	9543.16	4211.30	2168.52	4738.50	0.46
Kittler	1187.27	9614.03	5989.98	2796.59	5363.66	0.52
Moon & Spencer	498.44	1407.30	1118.98	308.25	1040.70	0.30

In terms of U_1 and U_2 , all other sky models (Table 4), except the Moon & Spencer, suggested the lowest uncertainty (CV = 0.01). This indicated that the Moon & Spencer sky model suggested higher uniformity inside the classroom. Meanwhile, Perez and Tregenza sky models showed similar CV values for both uniformity metrics ($U_1 = 0.54$ and $U_2 = 0.25$). A CV value greater than 0.1 indicated a lack of uniformity within the classroom. Furthermore, the Kittler sky model exhibited the highest CV value among the remaining sky models, with values of 1.13 for U_1 and 0.61 for U_2 .

Table 4 - Uniformity statistics for various sky models.

Sky Model	Min.	Max.	Median	σ	Mean	CV
U_1						
Perez	0.07	0.51	0.26	0.15	0.27	0.54
Tregenza	0.07	0.50	0.24	0.15	0.28	0.54
Kittler	0.04	0.66	0.08	0.24	0.22	1.13
Moon & Spencer	0.47	0.49	0.48	0.01	0.48	0.01
U_2						
Perez	0.28	0.74	0.50	0.13	0.52	0.25
Tregenza	0.29	0.72	0.51	0.13	0.53	0.25
Kittler	0.20	0.81	0.27	0.24	0.40	0.61
Moon & Spencer	0.65	0.68	0.66	0.01	0.66	0.01

Table 5 presents the ε statistics data for the illuminance values between the Perez sky model and the remaining sky models. The lowest mean ε was identified under the Tregenza sky model (mean $\varepsilon = 6.44\%$). The remaining sky models exhibited the increase of the mean ε values. Similarly, lowest mean ε values for U_1 (mean $\varepsilon = 8.15\%$) and U_2 (mean $\varepsilon = 3.78\%$) were discovered under Tregenza sky model. The rest of the sky models shows $U_1 > 50\%$ and $U_2 > 35\%$. Table 6 presents the complete ε statistics for uniformity values.

Table 5 - Average illuminance statistics for various sky models.

Metrics	Tregenza	Kittler	Moon & Spencer
Min. [%]	0.29	0.92	38.09
Max. [%]	19.51	299.59	94.28
Median [%]	4.55	47.60	75.23
Standard Deviation (σ)	5.52	80.39	14.95
Mean ε [%]	6.44	70.84	72.24

Table 6 - Relative error (ε) statistics for uniformity values. The comparison of Perez with Tregenza, Kittler and Moon & Spencer sky models.

Metrics	Tregenza		Kittler		Moon & Spencer	
	U_1	U_2	U_1	U_2	U_1	U_2
Min. [%]	0.22	0.09	28.11	5.25	0.28	0.27
Max. [%]	17.75	11.12	90.97	68.14	603.67	143.29
Median [%]	9.06	3.34	54.89	36.91	89.02	31.73
Standard Deviation (σ) [-]	4.99	2.63	16.43	16.82	178.16	38.20
Mean ε [%]	8.15	3.78	56.36	36.81	159.61	37.02

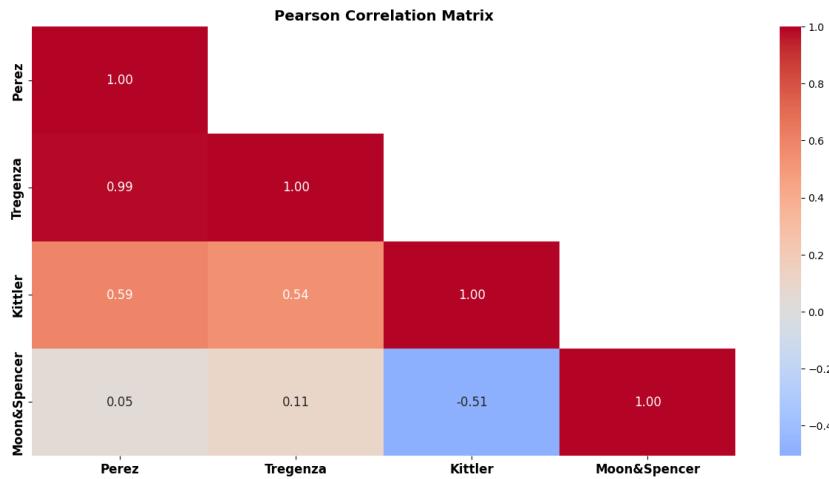


Fig. 4. Correlation matrix between evaluated sky models

Fig. 4 illustrates the correlation matrix between the evaluated sky models. The strongest positive correlation is observed between the Perez and Tregenza skies ($r = +0.99$). Additionally, a moderate positive correlation was identified between Perez and Kittler ($r = +0.59$) and between Tregenza and Kittler ($r = +0.54$). In contrast, a moderate negative correlation was observed between the Kittler and Moon & Spencer sky models, yielding a correlation coefficient of $r = -0.51$. Finally, the Perez and Tregenza sky models demonstrated a weak correlation with the Moon & Spencer sky model, with $r = +0.05$ and $r = +0.11$, respectively.

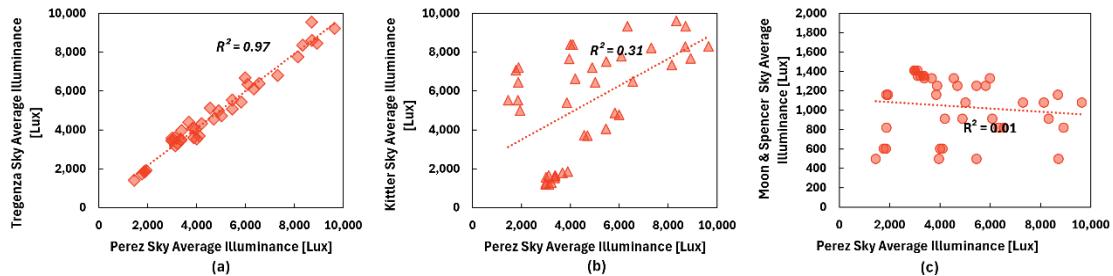


Fig. 5. Scatter plot between Perez sky model compared to (a) Tregenza, (b) Kittler and (c) Moon & Spencer sky models

The scatterplot (Fig. 5a) demonstrated that the Perez and Tregenza sky exhibited the strongest relationship in terms of aE ($R^2 = 0.97$). In contrast, the Kittler model exhibited a relatively weak relationship with the Perez sky ($R^2 = 0.31$, Fig. 5b). Finally, the Moon & Spencer model demonstrated a negligible relationship with the Perez sky model ($R^2 = 0.01$, Fig. 5c).

6. Discussion

This study has compared the daylight availability of a bilateral opening classroom typology under four different sky models, namely Perez, Tregenza, Kittler (CIE standard clear sky) and Moon & Spencer (CIE standard overcast sky). The average illuminance value is sensitive to the change of the sky models due to high uncertainty (high CV values). Furthermore, with regard to uniformity values, the Perez and Tregenza models suggest similar levels of uncertainty, whereas the Kittler model yields the highest uncertainty. Conversely, the Moon & Spencer model indicates the lowest uncertainty, suggesting that there is insignificant variation in illuminance values within the classroom with a bilateral opening typology.

In terms of the relative error, the highest error was found in the comparison between the Perez and Moon & Spencer sky models, both for their mean illuminance and uniformity values. Conversely, the lowest and highest relative error was discovered between the Perez compared to

Tregenza and the Perez compared to Kittler models, respectively. Consequently, the Perez model has the highest correlation to the Tregenza model as demonstrated in Figs. 4 and 5a.

Furthermore, the implication of these findings is that the illuminance values under Moon & Spencer sky model exhibits low values compared to other sky models. This type of sky model is generally utilized for daylight factor (DF) calculation as on the static metric for daylight performance and is still widely used worldwide (Abdelhakim, M., Lim, Y. W., & Kandar, 2019; Callejas et al., 2020; Dolnikova et al., 2020; Nasrollahi & Shokry, 2020; Pellegrino et al., 2015; Syahreza et al., 2018). This might lead to incorrect decision for daylight availability justification within the space, particularly in the context demonstrated in this study.

In addition, the high level of uniformity in the classroom under the Moon & Spencer sky model, as opposed to other sky models, could also lead to an inaccurate assessment of visual comfort due to underestimate values, such as potential for glare in a space, particularly in this study for the tropical climate in the classroom with the bilateral opening typology. Therefore, building designer must understand the implication of utilizing specific daylight metric for a daylight performance inside a space. Efforts such as adapting the static sky models to the local climate, to calculate the DF, as previously conducted in China (Aghimien & Li, 2022; Sun et al., 2021), could be a wise approach to adapt in the tropical climate in order to reduce the discrepancy.

The static daylight metrics employ a more generalized sky model, such as the Kittler and Moon & Spencer sky models, while the dynamic daylight metrics evaluate daylight based on sky models, such as the Perez and Tregenza. This study demonstrates that the Perez and Tregenza sky models are highly correlated, suggesting that the Tregenza sky model could serve as a representative sky model for annual daylighting performance evaluation, and it is relevant to the previous finding (Reinhart & Walkenhorst, 2001). However, for the annual contribution of direct sunlight into a room, a simulation can be performed under a modified Tregenza sky model, as previously proposed (Subramaniam & Mistrick, 2017). Furthermore, this study demonstrates that for tropical climates, the choice between static and dynamic metrics can have a significant impact on the design decision for daylighting performance within a space.

This finding suggests that in tropical climates with year-round daylight exposure, the bilateral opening classroom design utilizing static metrics, which has a direct relationship to, for instance, the Moon & Spencer sky, may lead to different design decisions when compared to the dynamic daylight evaluation, which is associated with the Perez and Tregenza sky in terms of illuminance values and uniformity conditions inside the space. Previous studies have suggested the potential benefits of employing a dynamic metric in the context of a bilateral opening typology classroom in tropical climates (Atthaillah et al., 2022a; Atthaillah et al., 2024a, 2024b). The recommendations set forth in previous studies are intended to facilitate a more accurate assessment of daylight availability and visual comfort within a classroom setting in the tropics. Alternatively, a more straightforward annual daylight metric calculation has been previously proposed with the objective of enabling the building designer to make an early prediction of annual daylight availability for a classroom in the tropics (Atthaillah et al., 2024b). The prediction model is based on the illuminance value under the Tregenza sky model.

Consequently, architects and building designers must be aware of this phenomenon, particularly when the objective is to design high-performance buildings. Nevertheless, the present study is constrained to a period of 40 critical hours of the year, which represents a relatively limited sample size. The findings of this study may facilitate the earlier identification of the optimal sky model for daylight simulation in tropical climates, thereby enhancing our comprehension of the discrepancies that may arise from the selection of different sky models in the tropics. A full-year evaluation may also be required in order to gain a more comprehensive understanding of the situation.

7. Conclusion

This study compares daylight availability in a bilateral opening classroom with different sky models, including the Perez, Tregenza, Kittler, and Moon & Spencer skies. The Perez and Tregenza models are comparable, and both skies exhibit similar levels of uncertainty. The Kittler model exhibits the highest uncertainty for its mean illuminance and uniformity values. The Moon

& Spencer model indicates the lowest uncertainty, suggesting insignificant variation in its uniformity values.

The relatively higher differences, both for average illuminance and uniformity values, between the Perez and Tregenza models and the rest of the sky models suggest that the static sky models are not comparable for daylight prediction in the tropics. Furthermore, the utilization of the static metric, despite its continued use by many scholars, in the tropics may result in incorrect design solutions for daylighting design. Consequently, it is imperative that building designers are aware of this situation to provide appropriate design solutions, particularly in the context of tropical classroom design.

Finally, while the static metric is employed on a continuous basis due to its simplicity in calculation, approaches such as adapting the static sky model to local climate, as demonstrated in studies conducted in China, could prove beneficial in improving the accuracy of daylight availability calculations. However, additional research is necessary to ascertain the suitability of this approach in tropical climates.

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